Theoretical Penetration of Hypervelocity Projectiles into Massive Targets

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A model of the penetration of a hypervelocity projectile into a massive target is formulated, and a quasi-theoretical technique for predicting the penetration is developed. It is assumed that 1) the projectile is significantly deformed immediately after impact by the initial shock wave and subsequent expansion waves, and 2) after the initial impact, the deformed projectile is retarded by a pressure on its face resulting from weak waves. The equation of projectile motion is integrated to obtain the depth of penetration. The resulting theoretical penetration is shown to be in agreement with experimental data.

Nomenclature

C = weak plastic wave speed, constant
c = weak plastic wave speed, variable
$\langle c \rangle$ = average wave speed
D = projectile diameter
K = penetration coefficient
L = projectile length
p = pressure on projectile-target interface
P = penetration depth from face of target
S = shock constant
t = time after impact
u = speed of projectile-target interface
U = shock speed
v = speed of projectile center of mass
V = impact speed
V_f = $(\sigma_{ut}/K\rho_t C_t)[1 + (\rho_t C_t/\rho_p C_p)]$
w = material speed
x = depth of projectile-target interface
z = depth from target surface
ρ = density
σ = material strength
Π = $K\rho_t C_t/\rho_p L_1$

Subscripts

m = incipient melting
p = projectile
t = target
u = ultimate strength
w = water
y = yield strength
0 = condition immediately after impact
1 = condition at the end of the first phase of penetration
i = condition at the end of the ith phase of penetration

Introduction

A SIZEABLE body of experimental data on the penetration of hypervelocity projectiles into massive targets is now available.[1-3] Several models of the penetration mechanism have been proposed,[4, 5] a number of analytical and empirical techniques for predicting penetration have been offered,[6-8] and detailed computations of the motion and stresses in the projectile and target after impact have been attempted.[9-11] Unfortunately, to date, no model, theory, or set of numerical calculations has been offered which is in general agreement with the body of experimental data. As Fuchs[6] has observed, "The problem of describing impact phenomena, in particular those resulting from hypervelocity impacts, is a complex one and has not yet been solved rigorously. Moreover, there is a good reason to presume that there never will be an ideal solution to this problem. . . ."

In the discussion that follows, a model and theory of hypervelocity impact are proposed. The development of this model was guided by examination of experimental data. Thus the resulting analysis is, to this extent, quasi-theoretical or semi-empirical. The assumptions upon which this model is based are unconventional but are defended by physical arguments. The resulting theory, or quasi-theory, is far from ideal, but it is sufficient agreement with experiment to be useful as an aid in the interpretation and extrapolation of experimental data and as a tool for the solution of design problems involving hypervelocity impacts.

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Penetration Model

When a hypervelocity projectile strikes a massive target, shock waves are formed which propagate into the target and backward into the projectile, compressing the material that they pass through. This compression is relieved by expansion waves emanating from the intersection of the shock waves and the free surfaces. The projectile is shortened somewhat by this initial shock compression, and then it expands radially as the compression is relieved by expansion waves from the lateral surfaces of the projectile. The projectile does not return to its original length but retains its shortened length throughout the remainder of the penetration process. If the projectile material is brittle, it may fracture because of this radial expansion. Since the argument presented here is based on conservation of the impact momentum, it is not greatly affected by whether fracturing does or does not occur.

The pressure on the projectile-target interface is a maximum immediately after impact and then falls rapidly as the initial compression is relieved. The projectile is slowed as well as squashed by this pressure pulse. Subsequently, the projectile continues to penetrate into the target material, which has been heated and set in motion by the target shock. During this phase of the process, the interface pressure is much lower than initially, and it falls slowly as the projectile slows down. This variation of pressure can be seen in the pressure-field calculations of Bjork,[9] Riney,[10] and Walsh and Tillotson.[11]

During this second phase of the penetration process, the stresses in the target are transmitted by weak plastic deformation waves. Thus, the stresses should be proportional to $(\rho_t c_t u)$, where ρ_t and c_t are, respectively, the density and weak plastic wave speed of the target, and u is the speed of the projectile-target interface relative to the target. During most of this phase of the process, the stress should not be proportional to the dynamic pressure $\rho_t(u - w_t)^2$, where w_t is the speed of the target material. This pressure is characteristic of most aerodynamic problems, but it is not appropriate in this case, since the flow is extremely unsteady.

At very high impact speeds, the impact-induced stresses are much greater than the strength of the projectile material. In this case, as proposed by Collins and Kinard,[5] the projectile can be considered to be made up of a bundle of arbitrarily slender "rods" aligned with the impact velocity. Each rod in the cluster is slowed by the pressure on its face but transmits only negligible retarding force to its neighbors. The penetration depth of the original projectile should be approximately the same as that of the "central" rod of the cluster. With this assumption, the effect of projectile shape is eliminated from the penetration computation, but no information can be gained about the crater shape or volume, since these are obviously affected by the other rods of the cluster.

With this model of the process, the penetration can be computed by 1) calculating the projectile deformation and deceleration resulting from the initial shock wave, and 2) integrating the equation of motion of the central rod of the projectile, under the assumption that the retarding force is proportional to $(\rho_t c_t u)$.

Initial Shock and Deformation

The relationship between the shock speed U and the speed of the material behind the shock w has been measured for most materials of interest. This relationship can be adequately approximated[12] in the form

$$U = C + Sw \tag{1}$$

where C and S are constants characteristic of the material. From the condition that momentum must be conserved across the shock, it can be shown that the initial pressure on the projectile-target contact surface p_0 is given by

$$p_0 = \rho_t U_{t0} w_{t0} = \rho_t(C_t + S_t w_{t0})w_{t0} \tag{2}$$

and

$$p_0 = \rho_p U_{p0} w_{p0} = \rho_p(C_p + S_p w_{p0})w_{p0} \tag{3}$$

And, from the condition of continuity on the interface,

$$V = w_{t0} + w_{p0} \tag{4}$$

where V denotes the impact speed of the projectile relative to the target, and subscripts t, p, and 0 denote the target material, the projectile material, the initial condition immediately after impact, respectively.

Equations (2–4) can be solved simultaneously to determine w_{t0} and w_{p0}. However, this calculation is numerically cumbersome. It is usually more convenient to determine these quantities graphically, as illustrated by Fig. 1.

From the condition that mass must be conserved across the shock, it is found that the density of the projectile material just behind the initial shock ρ_{p0} is given by

$$\rho_{p0} = \frac{\rho_p}{1 - (w_{p0}/U_{p0})} \tag{5}$$

Correspondingly, the projectile is initially compressed to a new length L_1 given by

$$L_1 = [1 - (w_{p0}/U_{p0})]L \tag{6}$$

where L is the original projectile length.

It is assumed here that, because of radial expansion, the projectile does not return to its original length but rather retains its shortened or squashed length throughout the remainder of the penetration process. In this model it is assumed that projectile deformation during the second phase of penetration is either negligible or occurs too late in the process to affect the penetration significantly. This approximation may not be adequate in the case of extremely high impact speeds or for rod-like projectiles.

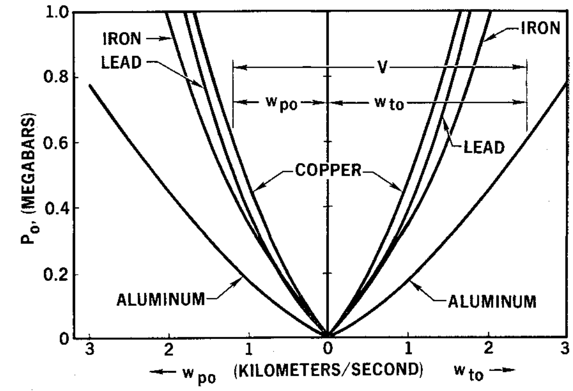

Fig. 1 Technique for graphical determination of w_{p0} and w_{t0}.

The pressure on the face of the central rod is constant for a brief instant after impact, and then it falls rapidly as the pressure is relieved by expansion waves from the lateral surfaces of the projectile. The duration of this initial retarding pulse t_1 is approximately

$$t_1 = D/2\langle C_p \rangle \tag{7}$$

where $\langle C_p \rangle$ denotes an average or effective wave speed and D is the diameter of the projectile.

As pointed out by Maiden,[13] the speed of the leading expansion wave C_{p0} can be computed, approximately, as

$$C_{p0} = U_{p0}\{0.49 + [1 - (w_{p0}/U_{p0})]^2\}^{1/2} \tag{8}$$

The last expansion wave moves much slower, since it must traverse the relaxed, shock-heated projectile material. The temperature of this material can be calculated by the method of Walsh and Christian.[14] The speed of plastic waves in hot or molten material is not available, but an adequate approximation to this wave speed can be made as follows. If the material is hot but not molten, the wave speed can be expected to vary as the square root of the slope of the stress-strain relationship in the plastic deformation regime. Thus, when better information is not available, it is assumed that the wave speed in the hot relaxed material C_{p1} is, approximately,

$$C_{p1} = C_p[(\sigma_{up1} - \sigma_{yp1})/(\sigma_{up} - \sigma_{yp})]^{1/2} \tag{9}$$

where σ_u is the ultimate strength, σ_y is the yield strength, and subscript one denotes the hot, relaxed material. If the material is molten, it is assumed that the plastic wave speed is given by

$$C_{p1} = C_w(\rho_w/\rho_t)^{1/2} \tag{10}$$

where C_w and ρ_w are the wave speed and density of water. The nature of the resulting relationship between the wave speed and the speed of the material behind the original shock wave is illustrated by Fig. 2.

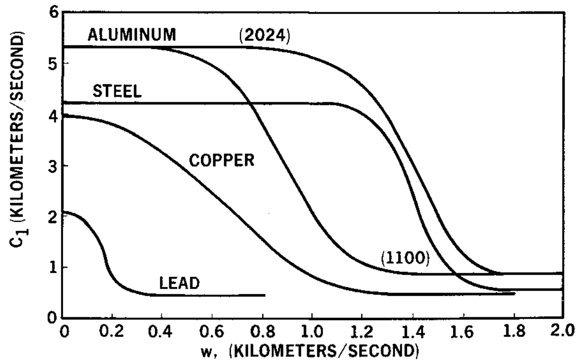

Fig. 2 Estimated relationship between plastic wave speed and the speed of the material behind the initial shock.

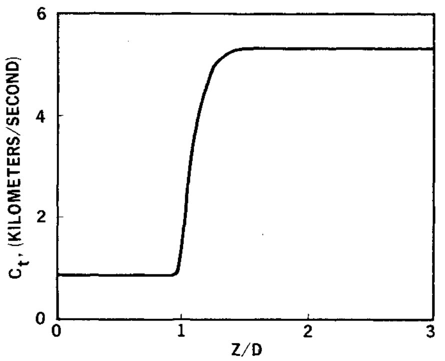

Fig. 3 Variation of plastic wave speed with depth for aluminum projectile striking aluminum target at V = 6.0 km/sec.

Since the speeds of the fastest and slowest waves are known approximately, it is assumed that the average wave speed $\langle C_p \rangle$ is given by

$$\langle C_p \rangle = \tfrac{1}{2}(C_{p0} + C_{p1}) \tag{11}$$

The change of momentum of the central rod caused by the initial retarding pulse can be expressed as

$$\rho_p L(V - V_1) = (\rho_p U_{p0} w_{p0}) t_1 \tag{12}$$

where V_1 is the speed of the projectile at time t_1, just after the initial impulse. By combining Eqs. (7) and (12),

$$V_1 = V - (D/L)(U_{p0}/2\langle C_p \rangle) w_{p0} \tag{13}$$

Correspondingly, at time $t = t_1$, the depth of the projectile-target interface below the original target surface x_1 is given by

$$x_1 = (w_{t0}/2\langle C_{p0} \rangle) D \tag{14}$$

It should be noted that Eq. (6) implies that, when the initial shock wave reflects from the rear surface of the projectile, it is immediately dissipated. As a consequence, if Eq. (13) yields a value of V_1 less than w_{t0}, it is inconsistent with the other assumptions. In this case, V_1 should be set equal to w_{t0}.

Plastic Weak-Wave Speed

In the discussion that follows, it is essential that the variation of the plastic weak-wave speed along the penetration centerline in the relaxed target material be known approximately. This variation can be estimated as follows.

It has been shown by Maiden[13] that the strength of the initial shock along the penetration centerline is constant when $z < 0.72D$, where z is the depth measured from the original face of the target. As the shock continues to advance, it is overtaken and weakened by expansion waves from the face of the target, assuming a nearly hemispherical shape. Thus, it is reasonable to expect w_t to fall inversely as the square of z when $z > 0.72D$. As a result, it is assumed that

$$U_t = C_t + S w_{t0} \tag{15}$$

where $z < 0.72D$; and, when $z > 0.72D$,

$$U_t = C_t + S w_{t0}(0.72D/z)^2 \tag{16}$$

Using the shock strength given by Eqs. (15) and (16), the temperature of the relaxed target material can be calculated by the method of Walsh and Christian,[14] and the weak-wave speed can be estimated by the method described in the preceding section.

Using these approximations, it is found that the wave speed is constant at a value denoted by C_{t1} when $x_1 < z < x_2$, where

$$x_2 = 0.72D \tag{17}$$

if the heated target material has some significant strength, or

$$x_2 = 0.72D(w_{t0}/w_{tm})^{1/2} \tag{18}$$

if $w_{t0} > w_{tm}$, where w_{tm} is the material speed when the target has no strength remaining or incipient melting occurs. When $z > x_2$, the wave speed rises rapidly to C_t in the undisturbed target material. This is illustrated for a typical case by Fig. 3.

Projectile Motion

After the initial retarding impulse, the stresses in the target are transmitted by weak plastic waves. Accordingly, it is assumed that the pressure at the stagnation point of the projectile-target interface, and thus on the central rod p, is given by

$$p = K\rho_t C_t u \tag{19}$$

where K is a characteristic "penetration coefficient," and c_t may vary considerably, as illustrated by Fig. 3.

Noting that p is the only significant retarding force acting, the motion of the central rod, when $t > t_1$, is given by

$$\rho_p L_1(dv/dt) = -K\rho_t c_t u \tag{20}$$

where v is the speed of the center of mass of the rod, and t is time measured from the initial impact. As a consequence of the previous assumption that deformation of the projectile is negligible when $t > t_1$,

$$v = u \tag{21}$$

Equations (20) and (21) can be combined to yield

$$du/dt = -(K\rho_t c_t/\rho_p L_1)u \tag{22}$$

The quantities K, ρ_p, and L_1 have been assumed to be nearly constant when $t > t_1$. Although the material just in front of the projectile is highly compressed by the initial shock, it is relaxed to nearly its normal density when $t > t_1$. Thus, ρ_t can be treated as constant in Eq. (22).

Since the variation of c_t, with depth from the original target surface, is known, Eq. (22) can now be integrated. It is convenient and adequate to carry out this integration over a

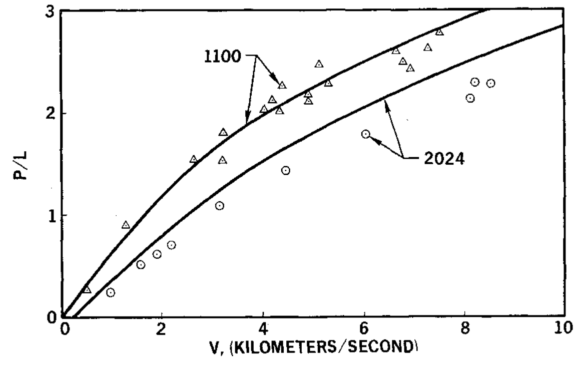

Fig. 4 Penetration of aluminum spheres into 1100[2] and 2024[3] aluminum.

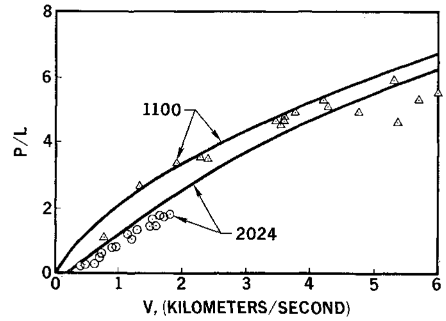

Fig. 5 Penetration of copper spheres into 1100[2] and 2024[1] aluminum.

series of intervals over which c_t can be considered to be locally constant.

Denoting the depth of penetration by x, so that

$$u = dx/dt \tag{23}$$

Eq. (22) can be integrated over the interval, $x_{i-1} < x < x_i$, to yield

$$u = V_{i-1}\, e^{-\Pi_i(t - t_{i-1})} \tag{24}$$

and

$$x = x_{i-1} + [(V_{i-1} - u)/\Pi_i] \tag{25}$$

where

$$\Pi_i = K\rho_t C_{ti}/\rho_p L \tag{26}$$

$$t_i = t_{i-1} - \frac{1}{\Pi_i} \ln\left[1 - \frac{\Pi_i(x_i - x_{i-1})}{V_{i-1}}\right] \tag{27}$$

and it has been assumed that c_t is constant at the value C_{ti} over the interval $x_{i-1} < x < x_i$.

From Eq. (25), the speed V_i is given in the convenient form

$$V_i = V_{i-1} - \Pi_i(x_i - x_{i-1}) \tag{28}$$

Penetration Depth

If the impact speed is very low so that $w_{p0} \ll C_p$ and little or no target heating occurs, then the penetration depth P is given by Eq. (28) as

$$P = (\rho_p L/K\rho_t C_t)(V - V_f) \tag{29}$$

where it has been assumed that the motion of the projectile ceases when p falls below the ultimate strength of the target material. V_f is given by Eqs. (2–4 and 19) as

$$V_f = (\sigma_{ut}/K\rho_t C_t)[1 + (\rho_t C_t/\rho_p C_p)] \tag{30}$$

Collins and Kinard[5] and Maurer and Rinehart[15] arrived at empirical penetration formulas quite similar to Eq. (29).

When the impact speed is large enough that projectile deformation cannot be neglected but target heating is negligible, the penetration is given by

$$P = \frac{\rho_p L_1}{K\rho_t C_t}(V - V_f) - \frac{w_{p0}}{2\langle C_p\rangle}\left(\frac{\rho_p U_{p0} L_1}{K\rho_t C_t L}\right) D + \frac{w_{t0}}{2\langle C_p\rangle} D \tag{31}$$

As the impact speed increases further, the penetration depth can be computed by estimating the variation of c_t with x, dividing x into intervals over which c_t is approximately constant, and applying Eq. (28) successively. If two intervals are used, the penetration depth is given by

$$P = \frac{\rho_p L_1}{K\rho_t C_{t3}}(V - V_f) - \frac{w_{p0}}{2\langle C_p\rangle}\frac{\rho_p U_{p0} L_1}{K\rho_t C_{t3}} D + \frac{w_{t0}}{2\langle C_p\rangle}\frac{C_{t2}}{Ct_3} D + x_2\left(1 - \frac{C_{t2}}{C_{t3}}\right) \tag{32}$$

If three intervals are used,

$$P = \frac{\rho_p L_1}{K\rho_t C_{t4}}(V - V_f) - \frac{w_{p0}}{2\langle C_p\rangle}\frac{\rho_p U_{p0} L_1}{K\rho_t C_{t4} L} D + \frac{w_{t0}}{2\langle C_p\rangle}\frac{C_{t2}}{C_{t4}} D + x_3\left(1 - \frac{C_{t3}}{C_{t4}}\right) + x_2\left(\frac{C_{t3} - C_{t2}}{C_{t4}}\right) \tag{33}$$

In the integration of Eq. (20), it has been assumed that the penetration coefficient K is not a function of the projectile speed or time, and it has been implied that K is not dependent on the projectile or target material or the projectile shape. Penetration data have been examined for several projectile shapes, for impact speeds up to 10 km/sec, and for 30 different combinations of projectile and target materials. In all cases this assumption appears to be justified, and K takes the value

$$K = \tfrac{1}{3} \tag{34}$$

when the impact speed is sufficiently high that the projectile deforms on impact. The reason for this seems to be that after deformation all projectiles assume a very blunt shape. The weak-wave pattern near the stagnation point of all blunt projectiles must be similar, and, thus, at sufficiently high impact speeds K is approximately the same for all projectiles.

The penetration of spherical projectiles given by this theory is compared with some of the experimental data available on Figs. 4–9. In this comparison the values of w_{p0}, w_{t0}, and U_{p0} were determined directly from experimental Hugoniot data, as illustrated by Fig. 1. The values of the plastic wave speed C were obtained by fitting Eq. (1) to experimental Hugoniot data. The following values of C were used: aluminum, C = 5.35 km/sec; copper, C = 3.97 km/sec; lead, C = 2.07 km/sec; and steel, C = 4.20 km/sec.

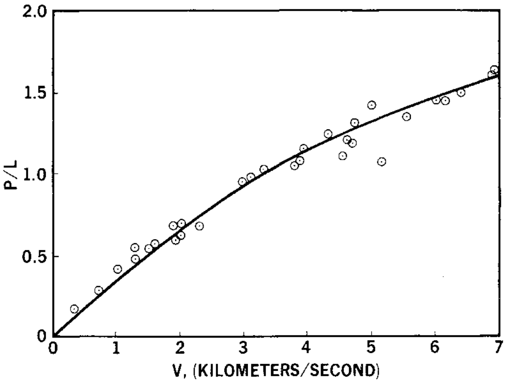

Fig. 6 Penetration of aluminum spheres into copper.[1, 2]

Concluding Discussion

The hypervelocity impact theory that has been developed indicates the following:

1) Penetration should increase with the projectile impact momentum per unit frontal area, as suggested by Collins and Kinard,[5] and inversely with the acoustic impedance of the target.

2) Penetration is only weakly dependent on target strength, per se, but it is strongly dependent on the plastic wave speed of the target material which is closely related to the target strength properties.

3) As the impact speed increases, the diameter of the projectile and the high-temperature properties of the target have an increasingly important effect on penetration.

This theory has been found to be in reasonable agreement with the available experimental data for spherical and squat-cylindrical projectiles at impact speeds up to 10 km/sec. This theory cannot be applied to projectiles of very low length/

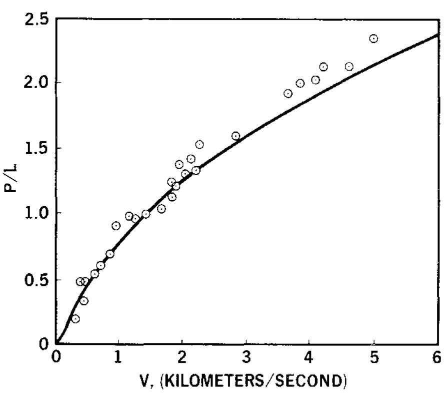

Fig. 7 Penetration of aluminum spheres into lead.[1]

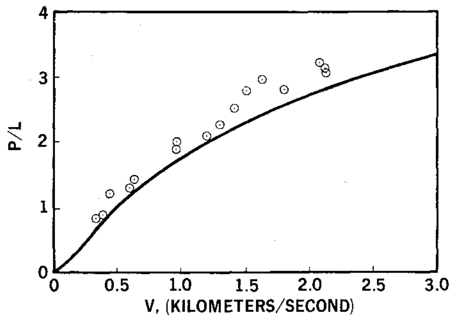

Fig. 8 Penetration of lead spheres into lead.[1]

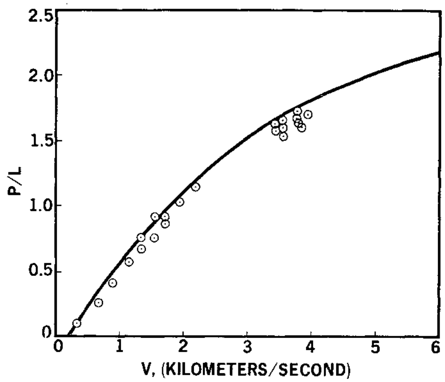

Fig. 9 Penetration of steel spheres into steel.[1]

diameter ratios because the expansion waves from the lateral surfaces of the projectile do not reach the center of the projectile until after penetration is largely complete, thus violating a key assumption of this model. This theory may also be inadequate at extremely high impact speeds, particularly for projectiles of very high length/diameter ratio, because projectile deformation during the second phase of penetration has been neglected.

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